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Tutorial 4 - Pigouvian Taxes and Pollution Permits II

A population of firms $\{1, \cdots, I\}$ pollutes the environment. Let q_i be the pollution level of the firm i and assume an homogenous pollutant emitted by the population of firms. Each firm has access to some pollution abatement technology. However this technology is costly to operate. If a firm i decides not to use this technology, its pollution level will be given by $\bar{q}_i$. By bearing a cost $C_i(q_i)$, the firm i can reduce the pollution level from $\bar{q}_i$ to q_i. Assume that:

Assumption C. 1 *: For each firm $i \in \{1, \cdots, I\}$, the cost function $C_i(q_i)$ is a class $\mathcal{C}^2$ function, $\Re_+ \to \Re_+$. $C_i(\bar{q}_i) = 0$: not exerting any pollution abatement effort costs nothing to the firm. Let $c_i(q_i) \equiv dC_i(q_i)/dq_i$ denote the marginal cost of abatement for firm i. Then $c_i(q_i)$ satisfies: $c_i(q_i) < 0$, $c_i(\bar{q}_i) = \underline{c}_i$, $\lim_{q_i \downarrow 0} c_i(q_i) = -\infty$, $c_i'(q_i) \equiv dc_i(q_i)/dq_i > 0$.*

An environmental agency tries to reach a global pollution objective $\bar{Q}$ by regulating the firms population pollution. Let Q be the total pollution by the firms population and assume to make sense of the problem that: $\sum_{i=1}^{I} \bar{q}_i > \bar{Q}$. That is: without pollution regulation, the firms would globally pollute too much with respect to the agency objective. The agency is also bothered by economic efficiency and it tries to set individual pollution standards for each firm i minimizing the total cost of the pollution abatement effort by the firms population.

- **Q 1:** Write the environmental agency problem as a constrained minimization problem (Hint: take care of the activity constraint $q_i \leq \bar{q}_i$!). Using the trick: $\min f(x) \equiv \max -f(x)$ transform this problem into a constrained maximization problem. Write down the corresponding Lagrangian and compute the first order conditions for optimality of an individual norms vector $\{q_1, \cdots, q_i, \cdots, q_I\}$ satisfying the global objective $Q \leq \bar{Q}$. Denote by λ the Lagrange multiplier associated to the global pollution constraint and by α_i the Lagrange multiplier associated to the pollution abatement activity constraint for each firm i.

- **Q 2:** Depending upon the severity of the pollution constraint $\bar{Q}$, does all firms have to make a pollution reduction effort ? Explain.

- **Q 3:** Let K be the subset of firms in $\{1, \cdots, I\}$ being active in reducing their pollution and $\bar{K} \equiv \{1, \cdots, I\} \setminus K$ be the subset of inactive firms in pollution abatement. What do you observe for the marginal cost of pollution abatement of the active firms in K? Give an economic interpretation of the result.

- **Q 4:** Suppose that the agency decides to implement a stricter global pollution norm $\bar{Q} < \bar{\bar{Q}}$. What will be the consequences over:

  - The level of the marginal abatement cost of the active firms?

  - The number of environmentally active firms?

  - The level of λ, the opportunity cost of the global pollution norm?

- **Q 5:** It appears that the global pollution level Q can be expressed as a function of λ, the marginal opportunity cost of the pollution norm. Let $Q(\lambda)$ be that function. Use the assumption $C.1$ and the first order conditions to deduce from the answer to Question Q.3 the sign of $dQ(\lambda)/d\lambda$. Infer from that a way to compute the optimal level of λ. Verify that this computation allows to determine the vector of the optimal individual norms $\{q_1^*, \cdots, q_i, \cdots, q_I^i\}$ to be imposed upon the different firms.

- **Q 6:** Instead of setting individual norms, the environmental agency decides to put in place a pollution tax. For the given level of pollution objective $\bar{\bar{Q}}$ what should be the level of the tax which would minimize the cost of compliance for the firms population? Do all firms reduce their pollution under the tax scheme? Explain.

- **Q 7:** Last, the agency, wondering about the firms heterogeneity, decides to put in place a pollution permits market system. The agency issues a number $\bar{Q}$ of permits and sell them to the firm through an auction process without free allocation of some permits. Then the firms are free to trade the permits on the market. What will be the level of the equilibrium price for the permits on the market?

- **Q 8:** If the agency decides to reduce the pollution allowance from $\bar{Q}$ to $\bar{Q}' < \bar{Q}$ by buying back on the market the corresponding amount of permits, do all firm willing to sell back some fraction of their permits? Explain.

- **Q 9:** Without loss of generality, rank the firms by increasing order of pollution levels without abatement effort, so that $\bar{q}_1 < \bar{q}_2 < \cdots < \bar{q}_i < \cdots, \bar{q}_I$. In order to circumvent the lobbying activity of the firms, the agency decides to grant a free pollution allowance to a subset J of the most polluting firms in case of no regulation. What will be the effect of such a free granting of permits over the equilibrium price of the permits?